CE 394K.2 Hydrology

Homework Problem Set #1

Due Thurs Feb 22

Problems in "Applied Hydrology"

1.1.6 Global average precip and evap

2.5.5 Water flow in a parking lot (not 2.2.5 as I had typed earlier, but if you've done 2.2.5 already that is fine).

2.6.2 Flow in a sand column

3.2.1 Atmospheric water properties

3.3.2 Velocity of a rain drop

3.4.4 Time series analysis of rainfall at a gage

$P_t = 4.5 \times 10N \times 1.02^{t-1980}$

The annual per capita water use is 6.8 m³/day = 2,482 x 10^{-9} km³/year. If per capita water use remains constant, the total water use for year t may be calculated as

$$P_t \times 2{,}482 \times 10^{-9}\ \text{km}^3/\text{year} = 4.5 \times 10^9 \times 1.02^{t-1980} \times 2{,}482$$

$$\times\ 10^{-9}\ \text{km}^3/\text{year} = 11{,}169 \times 1.02^{t-1980}\ \text{km}^3/\text{year}$$

Shortage of fresh water will occur when annual water use exceeds the 47,000 km³ of surface and subsurface runoff (Table 1.1.2 of the textbook) available for use on an annual basis, that is when

$$11{,}169 \times 1.02^{t-1980} > 47{,}000$$

or, after some algebra, when

$$t > 1980 + \ln (47{,}000/11{,}169) / \ln 1.02 = 2{,}052.6$$

Therefore, shortage will occur in the year 2053.

1.1.5.

The areas of land and ocean are (Table 1.1.2 of the textbook), respectively, 148,800,000 and 361,300,000 km². The annual precipitation values (Table 1.1.2 of the textbook) are 119,000 km³/year over the land and 458,000 km³/year over the ocean. The global average precipitation P may be calculated as

$$P = (119{,}000+458{,}000) / (148{,}800{,}000+361{,}300{,}000)\ \text{km/year}$$

$$= 1.131 \times 10^{-3}\ \text{km/year} = 113.1\ \text{cm/year}.$$

Annual evaporation equals 72,000 km³/year over the land and 505,000 km³/year over the ocean. The global average evaporation E may be calculated as

$$E = (72{,}000+505{,}000) / (148{,}800{,}000+361{,}300{,}000)\ \text{km/year} = 113.1\ \text{cm/year}$$

which is equal to the global average precipitation, as expected. Global evaporation and precipitation must be equal since, from the viewpoint of the atmospheric subsystem of the hydrologic cycle, evaporation and precipitation constitute the only input and output, respectively.

1.1.6.

The average precipitation is equal (Table 1.1.2 of the textbook) to 31 in/year over the land and 50 in/year over the ocean. The global average precipitation P may be calculated as a weighted average of these values,

$$P = (31 \times 148{,}800{,}000 + 50 \times 361{,}300{,}000) / (148{,}800{,}000+361{,}300{,}000)$$

$$= 44 \text{ in/year.}$$

The global average evaporation E may be calculated in a similar way,

$$E = (19 \times 148{,}800{,}000 + 55 \times 361{,}300{,}000) / (148{,}800{,}000+361{,}300{,}000)$$

$$= 44 \text{ in/year}$$

which is equal to the global average precipitation, as expected.

1.3.1.

The left side of the differential equation

$$K(dQ/dt) + Q(t) = I(t)$$

can be converted into an exact derivative multiplying by the factor $e^{t/K}$. The equation may thus be written as

$$Ke^{t/K} (dQ/dt) + e^{t/K} Q(t) = e^{t/K} I(t) \qquad \text{for } t \geq 0$$

so that

$$d(Ke^{t/K} Q)/dt = e^{t/K} I(t)$$

and, after integration

$$Ke^{t/K} Q = \int_0^t e^{\tau/K} I(\tau)\, d\tau + C \qquad \text{for } t \geq 0$$

where C stands for an integration constant. Then

$$Q = 1/K\, e^{-t/K} \{ \int_0^t e^{\tau/K} I(\tau)\, d\tau + C \} \qquad \text{for } t \geq 0$$

To determine the constant C, we may substitute $t = 0$ in the previous equation, obtaining

$$Q_0 = Q(0) = C/K$$

so that $C = KQ_0$. Therefore

$$Q = \int_0^t 1/K\, e^{(\tau-t)/K} I(\tau)\, d\tau + Q_0\, e^{-t/K} \qquad \text{for } t \geq 0 \qquad (1.3.1\text{-}1)$$

that Eq. (2.2.4-1) reduces to

$$\bar{I}_1 - \bar{Q}_1 = 0 \qquad (2.2.4\text{-}2)$$

The average annual inflow is

$\bar{I}_1$ = average annual runoff + precipitation on the reservoir

$= 0.33 \text{ m} \times (500 \times 10^9 \text{ m}^2) + 0.90 \text{ m} \times (1{,}700 \times 10^4 \text{ m}^2)$

$= 180{,}300{,}000 \text{ m}^3$

Notice that total precipitation on the reservoir is added to surface runoff, since the problem specifies the location of the reservoir at the basin outlet.

The average annual outflow is

$\bar{Q}_1$ = average annual evaporation + average annual withdrawal

$= 1.30 \text{ m} \times (1{,}700 \times 10^4 \text{ m}^2) + Q_d = 22{,}100{,}000 \text{ m}^3 + Q_d$

Then, substituting in Eq. (2.3.4-2)

$$\bar{I}_1 - \bar{Q}_1 = 180{,}300{,}000 - (22{,}100{,}000 + Q_d) = 0$$

so the average annual withdrawal is $Q_d = 158{,}200{,}000 \text{ m}^3\text{/year}$.

2.2.5.

As in Problem 2.2.4, the average annual inflow is

$\bar{I}_1$ = average annual runoff + precipitation on the reservoir

$= (13/12 \text{ ft}) \times (200 \times 5{,}280 \text{ ft}^2) + (35/12 \text{ ft}) \times (200 \times 5{,}280 \text{ ft}^2)$

$= 6{,}573{,}930{,}000 \text{ ft}^3$.

The average annual outflow is

$\bar{Q}_1$ = average annual evaporation + average annual withdrawal

$= (50/12 \text{ ft}) \times (200 \times 5{,}280 \text{ ft}^2) + Q_d = 762{,}300{,}000 \text{ ft}^3 + Q_d$

Then

$$\bar{I}_1 - \bar{Q}_1 = 6{,}573{,}930{,}000 - (762{,}300{,}000 + Q_d) = 0$$

so the average annual withdrawal is

$Q_d = 5{,}811{,}630{,}000 \text{ ft}^3\text{/year} = 133{,}416.7 \text{ ac.ft/year}$.

$= S_0 = 0.01$ for uniform flow

$$v = (1.49/n)\ R^{2/3}\ S_f^{1/2} = (1.49/0.035) \times 2.75^{2/3} \times 0.01^{1/2} \text{ ft/s}$$
$$= 8.35 \text{ ft/s}$$

The flow rate is $Q = v\ A = 8.35 \times 327$ cfs $= 2{,}731$ cfs. The criterion for fully turbulent flow is calculated from Eq. (2.5.9a) in the textbook

$$n^6\ (RS_f)^{1/2} = 0.035^6\ (2.75 \times 0.01)^{1/2} = 3.05 \times 10^{-10}$$

which is larger than 1.9×10^{-13} so the criterion is satisfied and Manning's equation is applicable.

2.5.4.

The area of the channel is $A = (30 + 3 \times 1) \times 1\ m^2 = 33\ m^2$. The wetted perimeter is

$$P = 30 + 2 \times 1 \times (1^2 + 3^2)^{1/2}\ m = 33.16\ m$$

so the hydraulic radius is $R = A/P = 33/33.16 = 0.995$ m.

The flow velocity is given by Manning's equation with $n = 0.035$ and $S_f = S_0 = 0.01$ for uniform flow

$$v = (1/n)\ R^{2/3}\ S_f^{1/2} = (1/0.035) \times 0.995^{2/3} \times 0.01^{1/2}\ m/s = 2.85\ m/s$$

The flow rate is $Q = v\ A = 2.85 \times 33\ m^3/s = 94\ m^3/s$. The criterion for fully turbulent flow is calculated from Eq. (2.5.9b) in the textbook

$$n^6\ (RS_f)^{1/2} = 0.035^6\ (0.995 \times 0.01)^{1/2} = 1.83 \times 10^{-10}$$

which is larger than 1.1×10^{-13} so the criterion is satisfied and Manning's equation is applicable.

2.5.5.

Shallow flow over a parking lot is equivalent to flow in an infinite width channel; in this case, the flow can be analyzed in a portion of channel of unit width. For a flow depth of 1 in, the area of this channel portion is $A = 1/12 \times 1\ ft^2 = 0.083\ ft^2$. The wetted perimeter corresponds only to the channel bottom, $P = 1$ ft, so the hydraulic radius is $R = A/P = 0.083/1$ ft $= 0.083$ ft, equal to the flow depth.

ne flow velocity is given by Manning's equation with $n = 0.015$ and $S_f = S_C$).5 % = 0.005 for uniform flow

$$. = (1.49/n)\ R^{2/3}\ S_f^{1/2} = (1.49/0.015) \times 0.083^{2/3} \times 0.005^{1/2} \text{ ft/s} =$$

$= 1.34$ ft/s

The flow rate per unit width of channel is Q = v A = 1.34 x 0.083 cfs/ft = 0.1112 ~~2.731~~ cfs/ft. The criterion for fully turbulent flow is calculated from Eq. (2.5.9a) in the textbook

$$n^6 \, (RS_f)^{1/2} = 0.015^6 \, (0.083 \times 0.005)^{1/2} = 2.32 \times 10^{-13}$$

which is larger than 1.9×10^{-13} so the criterion is satisfied and Manning's equation is applicable.

2.5.6.

For a flow depth of 1 cm, the area of a unit width channel portion is A = 0.01 x 1 m² = 0.01 m². The wetted perimeter corresponds only to the channel bottom, P = 1 m, so the hydraulic radius is R = A/P = 0.01/1 m = 0.01 m, equal to the flow depth.

To check whether Manning's equation is applicable, the criterion for fully turbulent flow is calculated from Eq. (2.5.9b) in the textbook

$$n^6 \, (RS_f)^{1/2} = 0.015^6 \, (0.01 \times 0.005)^{1/2} = 0.81 \times 10^{-13}$$

which is smaller than 1.1×10^{-13} so the criterion is not satisfied (although the flow is very close to fully turbulent); Manning's equation is not applicable and the Darcy-Weisbach equation should be used instead.

The relative roughness ε is computed using Eq. (2.5.15) from the textbook, as a function of the hydraulic radius R = 0.01 m and the Manning's coefficient n = 0.015, with the factor $\Phi = 1$ for SI units. Then

$$\varepsilon = 3 \times 10^{-\Phi R^{1/6}/[4n(2g)^{1/2}]} = 0.054$$

The flow velocity v and the Darcy-Weisbach friction factor f have to be calculated in an iterative fashion. For a given value of v, the Reynolds number Re can be computed using Eq. (2.5.10) from the textbook. For a value of the kinematic viscosity $\nu = 10^{-6}$ m²/s, we have

$$Re = 4vR/\nu = 4v \times 0.01/10^{-6} = 40{,}000 \, v \qquad (2.5.6\text{-}1)$$

The friction factor f can then be updated using the modified Moody diagram in Fig. 2.5.1 of the textbook or the Colebrook-White equation (Eq. 2.5.13 from the textbook). In this case the latter method will be preferred since it is more accurate and easier to implement in a computer code. For flow in the transition zone,

$$1/\sqrt{f} = -2 \log_{10}[\varepsilon/3 + 2.5/Re \times 1/\sqrt{f}] =$$

$$= -2 \log_{10}[0.054/3 + 2.5/Re \times 1/\sqrt{f}] \qquad (2.5.6\text{-}2)$$

$$q = KS_f = 10 \times 0.01 = 0.1 \text{ cm/s} = 86.4 \text{ m/d}$$

and

$$v_a = q/\eta = 86.4/0.3 = 288 \text{ m/d} = 3 \times 10^{-3} \text{ m/s}$$

Notice that the velocity in the 1 mm capillary tube is of the same order of magnitude than the velocity of flow through the gravel.

2.6.2.

The length of the conduit is $L = 10$ m and the pressure head is $\Delta h = 0.5$ m, so the friction slope is $S_f = \Delta h/L = 0.05$. The kinematic viscosity of water is, at 20° C, $\nu = 10^{-6}$ m²/s. The rate of flow is given by Darcy's law (Eq. 2.6.4 of the textbook)

$$Q = KAS_f$$

The hydraulic conductivity K can be calculated from the Hagen-Poiseulle equation (Eq. 2.6.3 of the textbook) with area A = 2 m² and equivalent diamet.. $D = 0.01$ mm $= 10^{-5}$ m for fine sand, yielding

$$K = gD^2/(32\nu) = 9.81 \times (10^{-5})^2/(32 \times 10^{-6}) = 3 \times 10^{-5} \text{ m/s}$$

Therefore, the flow rate is

$$Q = 3 \times 10^{-5} \times 2 \times 0.05 = 3 \times 10^{-6} \text{ m}^3/\text{s} = 0.26 \text{ m}^3/\text{d}$$

(10.8 l/hr)
3.0×10^{-6} m³/s

(1) Medium	(2) Hydraulic Conductivity K (cm/s)	(3) Porosity η	(4) Darcy velocity q (m/d)	(5) Actual velocity v (m/d)	(6) Travel time T (days)
Gravel	10	0.3	8.64	28.8	3.5
Silt	10^{-4}	0.4	8.64×10^{-4}	2.16×10^{-3}	46,296
Clay	10^{-7}	0.5	8.64×10^{-7}	1.73×10^{-6}	5.78×10^6

Table 2.6.3. Travel time of water to the stream.

3.2.1.

The saturated vapor pressure at T = 25 °C is given by Equation (3.2.9) of the textbook

$$e_s = 611 \exp[17.27T/(237.3+T)] = 611 \exp[17.27 \times 25/(237.3 + 25)]$$

so e_s = ~~2598~~ 3169 Pa. The actual vapor pressure, e, is calculated by the same method substituting the dew point temperature T_d = 20 °C for T

$$e = 611 \exp[17.27T_d/(237.3+T_d)] = 611 \exp[17.27 \times 20/(237.3 + 20)]$$

so e = ~~1984~~ 2339 Pa.

The relative humidity, from Equation (3.2.11) of the textbook, is

$$R_h = e/e_s = \sout{1984/2598}\ 2339/3169 = \sout{0.76}\ 0.74$$

and the specific humidity is given by Equation (3.2.6) of the texbook, with air pressure p = 101.1 x 10³ Pa

$$q_v = 0.622\, e/p = 0.622 \times \sout{1984}\ 2339/(101.1 \times 10^3) = \sout{0.012}\ 0.0144$$

The gas constant for air, R_a, is given by Equation (3.2.8) of the texbook

$$R_a = 287\,(1 + 0.608\, q_v) = 287\,(1 + 0.608 \times 0.01\sout{2}4) = 289\ J/(kg.{}^oK)$$

and the air density is calculated from the ideal gas law (Equation 3.2.7 of the textbook) with temperature T = 273 + 25 = 298 °K, so that

$$\rho_a = p/(R_a T) = 101.1 \times 10^3/(289 \times 298) = 1.17\ kg/m^3$$ ✓

3.2.2.

The temperature T_2 at elevation z_2 = 1500 m is given by Equation (3.2.16) of the textbook with T_1 = 25°C, z_1 = 0, and lapse rate α = 9 °C/km, so that

$$T_2 = T_1 - \alpha\,(z_2 - z_1) = 25 - 9 \times 10^{-3}\,(1500 - 0) = 11.5\ {}^oC = 284.5\ {}^oK$$

This temperature is below the dew point temperature for surface conditions, T_d = 20 °C, so the vapor pressure at 1500 m elevation corresponds to the saturated vapor pressure given by Equation (3.2.9) of the textbook

$$e_s = 611 \exp[17.27T/(237.3+T)] = 611 \exp[17.27 \times 11.5/(237.3 + 11.5)]$$

so $e = e_s$ = 1357 Pa.

The air pressure p_2 at z_2 = 1500 m is given by Equation (3.2.15) from the textbook, with p_1 = 101.1 kPa, so that

$$p_2 = p_1\,(T_2/T_1)^{g/(\alpha R_d)} = 101.1 \times (284.5/298)^{9.81/(0.009 \times 289)} = 84.9\ kPa$$

Surface Temperature (^oC)	Precipitable Water (mm)
0	9.95
10	20.77
20	41.01
30	77.03
40	138.39

Table 3.2.6. Variation of precipitable water depth with surface temperature.

3.3.2.

From Equation (3.3.4) of the textbook, the terminal velocity v_t of a falling raindrop of diameter D = 2 mm = 0.002 m, with C_d = 0.517 from Table 3.3.1 of the textbook, is

$$v_t = [4gD/(3C_d)\ (\rho_w/\rho_a - 1)]^{1/2}$$

$$= [4 \times 9.81 \times 0.002/(3 \times 0.517)\ (998/1.20 - 1)]^{1/2} = \underline{6.48 \text{ m/s}}$$

this is the drop velocity relative to the surrounding air. If the air is rising with velocity v_a = - 5 m/s, the absolute velocity of the drop is

$$v_{rel} = v_t + v_a = 6.48 - 5 = \underline{1.48 \text{ m/s}}$$

and the drop is falling.

For drop diameter D = 0.2 mm = 0.0002 m, with C_d = 4.2, the terminal velocity can be calculated similarly, resulting v_t = 0.72 m/s and

$$v_{rel} = v_t + v_a = 0.72 - 5 = - \underline{4.28 \text{ m/s}}$$

and the drop is rising.

3.3.3.

Three vertical forces act on a falling raindrop: a gravity force F_w due to its weight, a buoyancy force F_b due to the air displaced by the drop and a drag force F_d caused by the friction between the drop and the surrounding air. If v is the vertical fall velocity of the drop of mass m, from Newton's law

$$m\ dv/dt = F_w - F_b - F_d \tag{3.3.3-1}$$

The maximum rainfall depth recorded in 10, 20 and 30 min. intervals is found by computing the running totals in Cols. (4), (5) and (6) of Table 3.4.3, respectively, through the storm, then selecting the maximum value of the corresponding series, as shown in Table 3.4.3. For example, for a 30 minute time interval, the maximum 30 minute depth is 1.16 in, recorded between 5 and 35 min. The rainfall intensity (depth divided by time) corresponding to this depth is 1.16 in/0.5 hr = 2.32 in/hr. This value is less than 60 % of the 30 min intensity experienced at gage 1-Bee for the same storm (see Table 3.4.1 from the textbook).

3.4.4.

The computations follow those in Problem 3.4.3. and are summarized in Table 3.4.4. The storm hyetograph is shown in Fig. 3.4.4(a). The cumulative rainfall hyetograph, or rainfall mass curve, is obtained in Col. (3) of Table 3.4.4, and plotted in Fig. 3.4.4(b).

The maximum rainfall depth or intensity (depth divided by time) recorded in 10, 30, 60, 90 and 120 min. intervals is found by computing the running totals in Cols. (4)-(8) of the table, through the storm, then selecting the maximum value of the corresponding series, as shown in Table 3.4.4. These intensities are about 60-70 % of the intensities observed at gage 1-Bee for the same storm (see Table 3.4.1 from the textbook), which experienced more severe rainfall.

3.4.5.

(a) Arithmetic mean method. Raingages numbers 1, 4, 6 and 8 are located outside the watershed and will not be considered in the computation of the arithmetic mean. The areal average rainfall is, therefore,

$$\bar{P} = (P_2 + P_3 + P_5 + P_7 + P_9)/5$$

$$= (59 + 41 + 105 + 60 + 81)/5 = 69.2 \text{ mm}$$

(b) Thiessen method. The Thiessen polygon network is shown in Fig. 3.4.5-1. The areas assigned to each station are shown in Col. (3) of Table 3.4.5-1. The watershed area is $A = 125$ km^2 and the areal average rainfall is given by Eq. (3.4.1) of the textbook

$$\bar{P} = 1/A \sum_{j=1}^{J} A_j P_j = 8746/125 = 70.0 \text{ mm}$$

(c) Isohyetal method. The isohyetal map is shown in Fig. 3.4.5-2. The average rainfall is found by adding the weighted rainfall values in Col. (4) of Table 3.4.5-2,

$$\bar{P} = 1/A \sum_{j=1}^{J} A_j P_j = 8638/125 = 69.1 \text{ mm}$$

Time	Rain (in)	Cum Rain	10min	30min	60min	90in	120min
0	0	0	0	0	0	0	0
5	0.09	0.09	0	0	0	0	0
10	0	0.09	0.09	0	0	0	0
15	0.03	0.12	0.03	0	0	0	0
20	0.13	0.25	0.16	0	0	0	0
25	0.1	0.35	0.23	0	0	0	0
30	0.13	0.48	0.23	0.48	0	0	0
35	0.21	0.69	0.34	0.6	0	0	0
40	0.37	1.06	0.58	0.97	0	0	0
45	0.22	1.28	0.59	1.16	0	0	0
50	0.3	1.58	0.52	1.33	0	0	0
55	0.2	1.78	0.5	1.43	0	0	0
60	0.1	1.88	0.3	1.4	1.88	0	0
65	0.13	2.01	0.23	1.32	1.92	0	0
70	0.14	2.15	0.27	1.09	2.06	0	0
75	0.12	2.27	0.26	0.99	2.15	0	0
80	0.16	2.43	0.28	0.85	2.18	0	0
85	0.14	2.57	0.3	0.79	2.22	0	0
90	0.18	2.75	0.32	0.87	2.27	2.75	0
95	0.25	3	0.43	0.99	2.31	2.91	0
100	0.48	3.48	0.73	1.33	2.42	3.39	0
105	0.4	3.88	0.88	1.61	2.6	3.76	0
110	0.39	4.27	0.79	1.84	2.69	4.02	0
115	0.24	4.51	0.63	1.94	2.73	4.16	0
120	0.41	4.92	0.65	2.17	3.04	4.44	4.92
125	0.44	5.36	0.85	2.36	3.35	4.67	5.27
130	0.27	5.63	0.71	2.15	3.48	4.57	5.54
135	0.17	5.8	0.44	1.92	3.53	4.52	5.68
140	0.17	5.97	0.34	1.7	3.54	4.39	5.72
145	0.14	6.11	0.31	1.6	3.54	4.33	5.76
150	0.1	6.21	0.24	1.29	3.46	4.33	5.73
Max Depth	0.48		0.88	2.36	3.54	4.67	5.76
Intensity (in/hr)	5.76		5.28	4.72	3.54	3.11	2.88

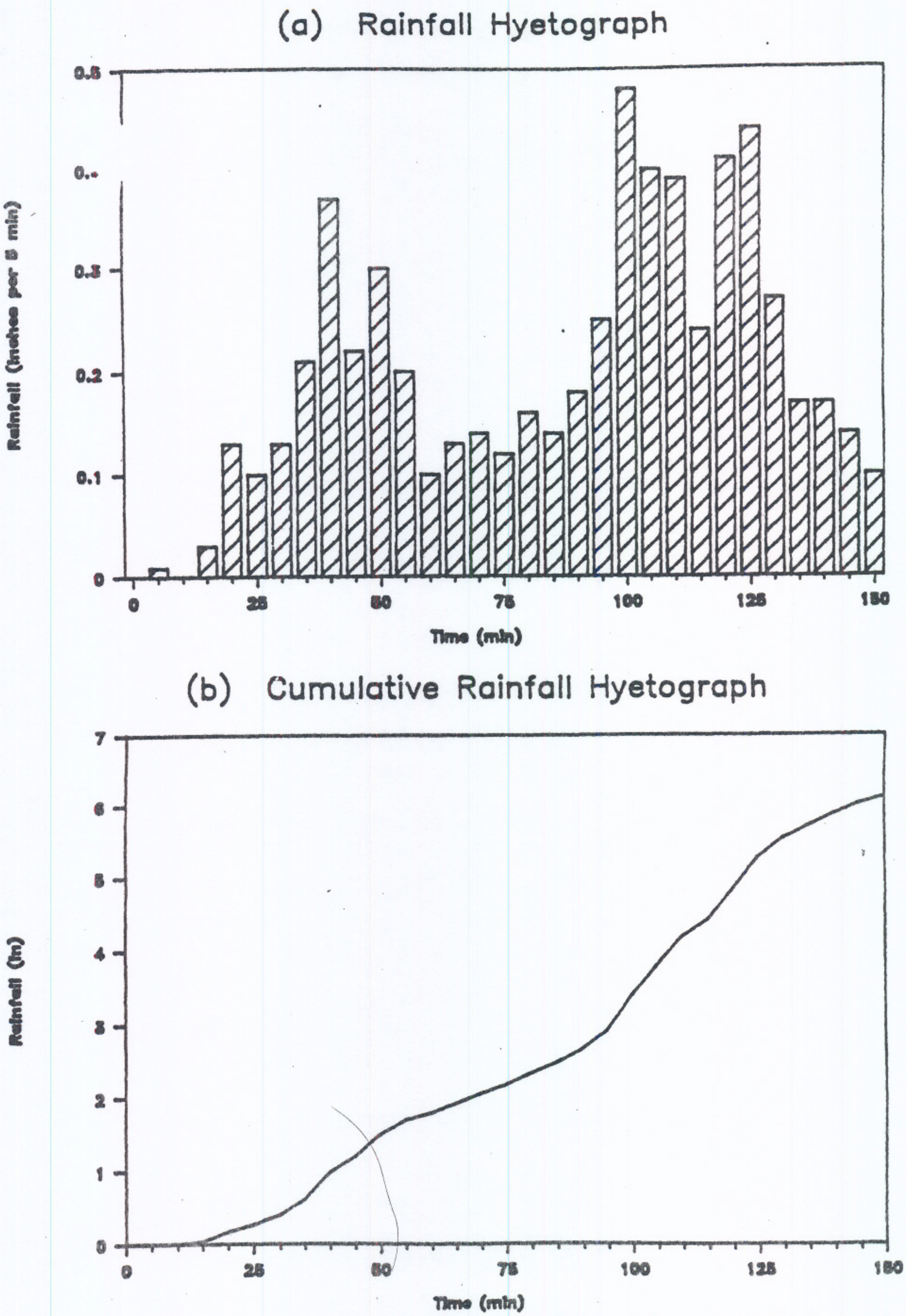

Fig. 3.4.4. Rainfall hyetograph (in 5 min. increments) and cumulative rainfall hyetograph at gage 1-WLN for the storm of May 24-25, 1981 in Austin, Texas.